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Fixed Beamformer Design Using Polynomial Eigenvalue Decomposition

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Summary

- Eigenvector generated by PEVD can be interpreted as a spatio-temporal filter and used to design fixed beamformers
- Suited for arbitrary arrays
- Lightweight with an average filter length of 114
- Fixed PEVD beamformers performs comparably to data-dependent MVDR and LCMV for separation of sources closely spaced by 5°

Multichannel Signal Model

Multichannel model (P sources, Q sensors):

$$x_q(n) = \sum_{p=1}^{P} h_{p,q}(n) * s_p(n) + v_q(n)$$

From Q microphones: $\mathbf{x}(n) = [x_1(n), x_2(n), \ldots, x_Q(n)]^T \in \mathbb{R}^Q$

Assuming stationarity, space-time covariance matrix:

$$\mathbf{R}(\tau) = \mathbb{E}[\mathbf{x}(n)\mathbf{x}^T(n-\tau)]$$

The z-transform, also denoted by $\mathbf{R}(\tau) \circ\!\!-\!\!\bullet \mathcal{R}(z)$, is:

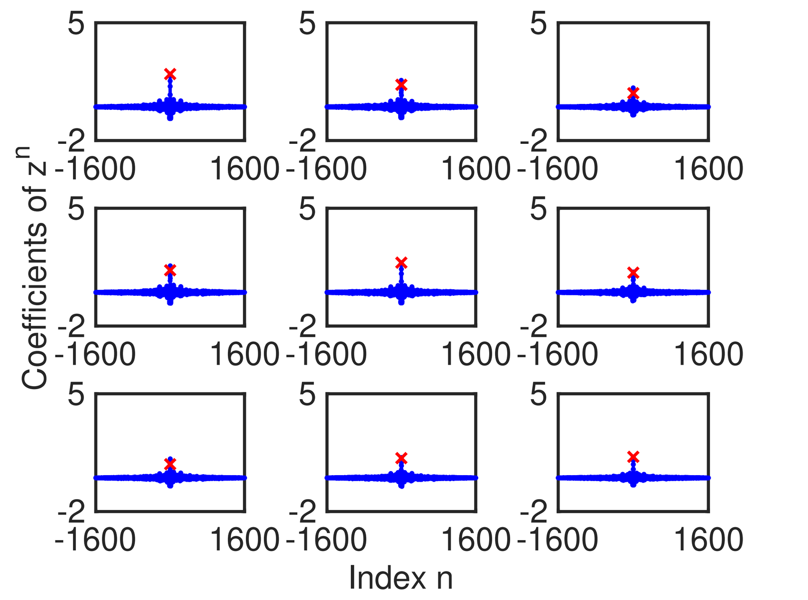
$$\mathcal{R}(z) = \sum_{\tau=-\infty}^{\infty} \mathbf{R}(\tau) z^{-\tau}$$

The polynomial EVD (PEVD) of para-Hermitian $\mathcal{R}(z) \in \mathbb{C}^{Q\times Q}$ is [1]:

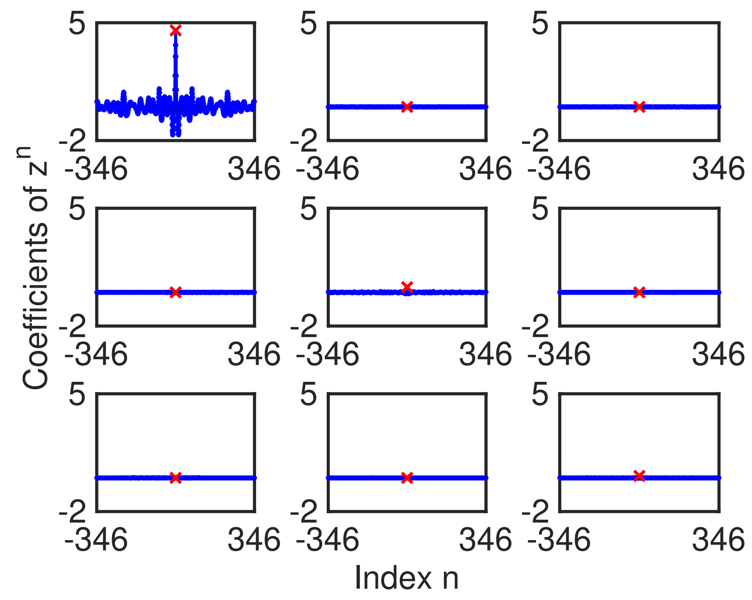
$$\mathcal{R}(z) = \mathcal{U}(z)\,\mathbf{\Lambda}(z)\,\mathcal{U}^P(z) \Leftrightarrow \mathbf{\Lambda}(z) = \mathcal{U}^P(z)\,\mathcal{R}(z)\,\mathcal{U}(z)\,, \quad (1)$$

with eigenvalues, $\mathbf{\Lambda}(z)$, and paraunitary eigenvectors, $\mathcal{U}(z) \in \mathbb{C}^{Q\times Q}$.

Example of $\mathcal{R}(z)$



Eigenvalue, $\mathbf{\Lambda}(z)$



Fixed Beamformer Design Using PEVD

Rewriting as $\mathbf{H}(n) \circ\!\!-\!\!\bullet \mathcal{H}(z) \in \mathbb{C}^{P\times Q}$, where each element is $h_{p,q}(n)$:

$$\mathcal{R}_{\mathbf{x}}(z) = \mathcal{H}^P(z)\,\mathcal{R}_{\mathbf{s}}(z)\,\mathcal{H}(z) + \sigma_v^2\,\mathbf{I}\,, \quad (2)$$

with spatially and temporally white noise $\mathbf{v}(n)$ of equal power σ_v^2.

With i.i.d. source signals and each drawn from $\mathcal{N}(0,1)$, $\mathcal{R}_{\mathbf{s}}(z) = \mathbf{I} \in \mathbb{C}^{P\times P}$.
Applying PEVD in (1) to (2) and rearranging:

$$\mathbf{\Lambda}(z) - \sigma_v^2\,\mathbf{I} = \mathcal{U}^P(z)\,\mathcal{H}^P(z)\,\mathcal{H}(z)\,\mathcal{U}(z)\,.$$

Diagonalization $\implies$ $\mathcal{U}(z)$ spatially decorrelate the acoustic channels $\mathcal{H}(z)$.

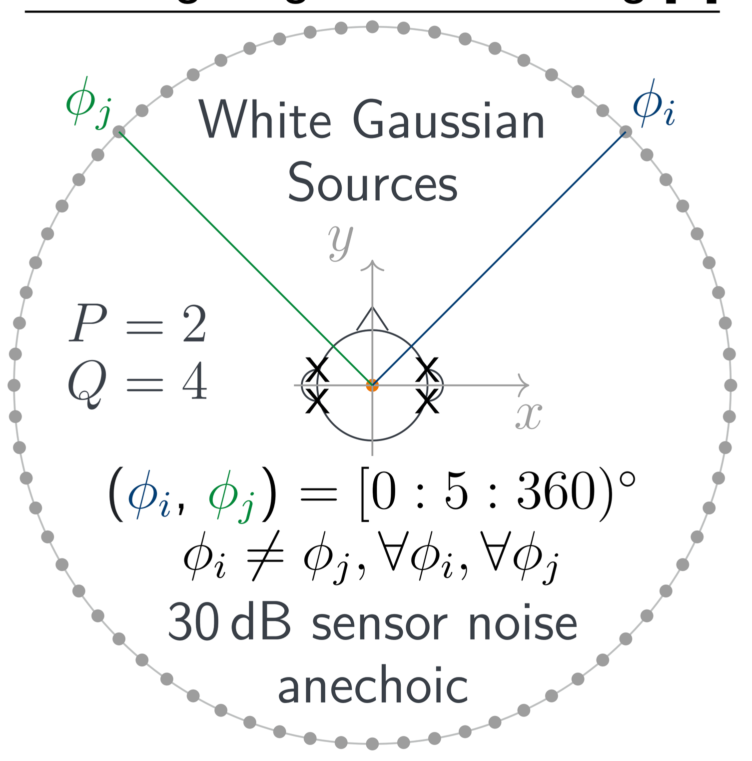
By evaluating on the unit circle, beampattern response at frequency Ω:

$$\mathbf{B}(\phi,\Omega) = [\mathcal{U}^P(z)\boldsymbol{a}_\phi(z)]\Big|_{z=e^{j\Omega}}\,,$$

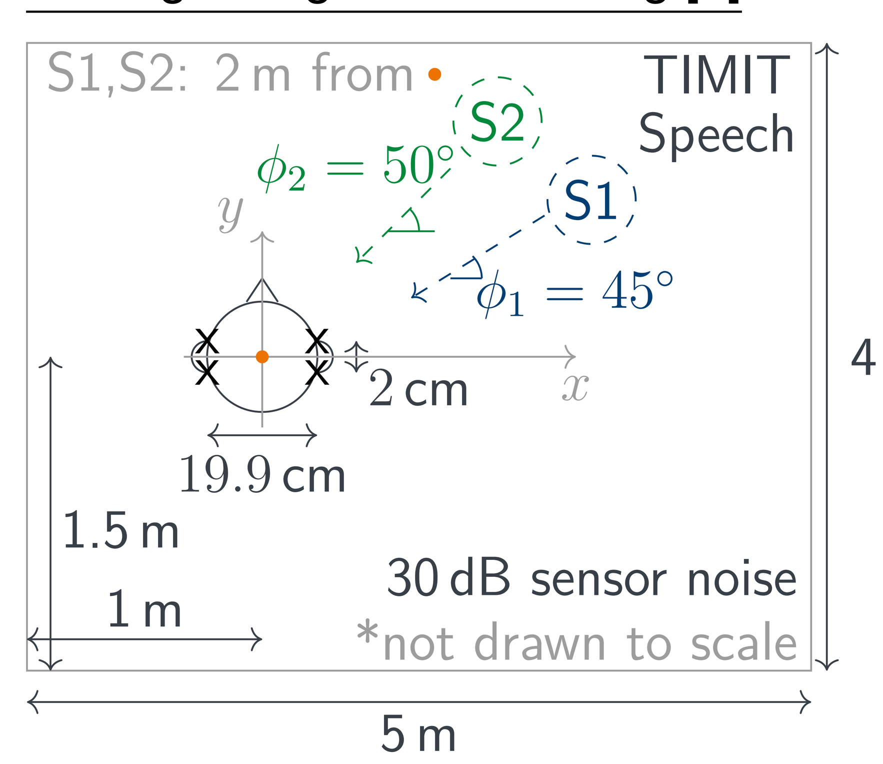
where $\mathbf{a}_\phi(n) \circ\!\!-\!\!\bullet \boldsymbol{a}_\phi(z) \in \mathbb{C}^Q$ is the broadband steering vector using array geometry and the qth element is $a_q(n) = \mathrm{sinc}(nT_s - \Delta\tau_q)$ with sampling period T_s and relative time delay $\Delta\tau_q$.

Training and Testing of PEVD Fixed Beamformers

Training IR generated using [2]

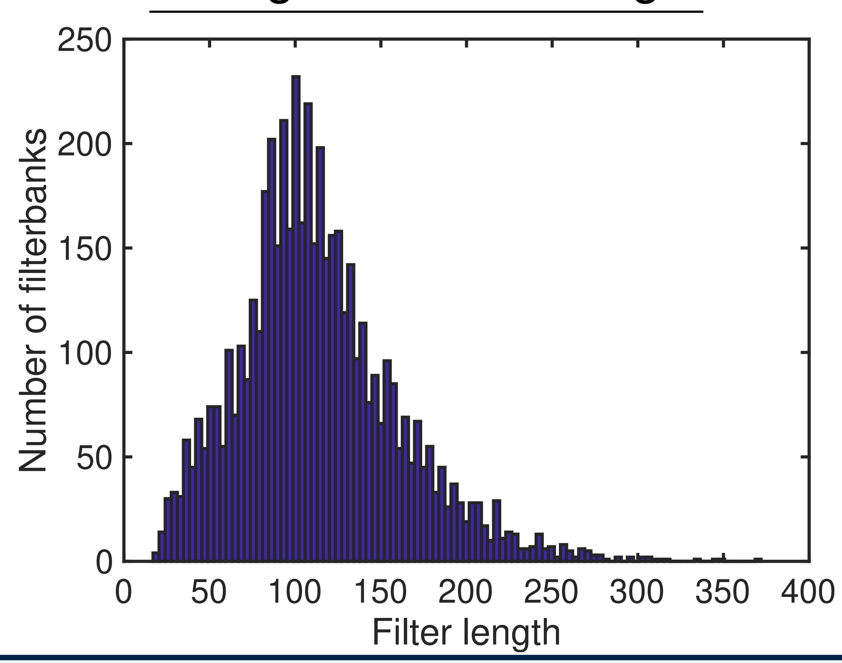


Testing RIR generated using [3]



Training and Evaluation

Histogram of filter length



Comparative beamformers
1. MVDR
2. LCMV
3. Fixed PEVD

Evaluation measures
- STOI
- SIR

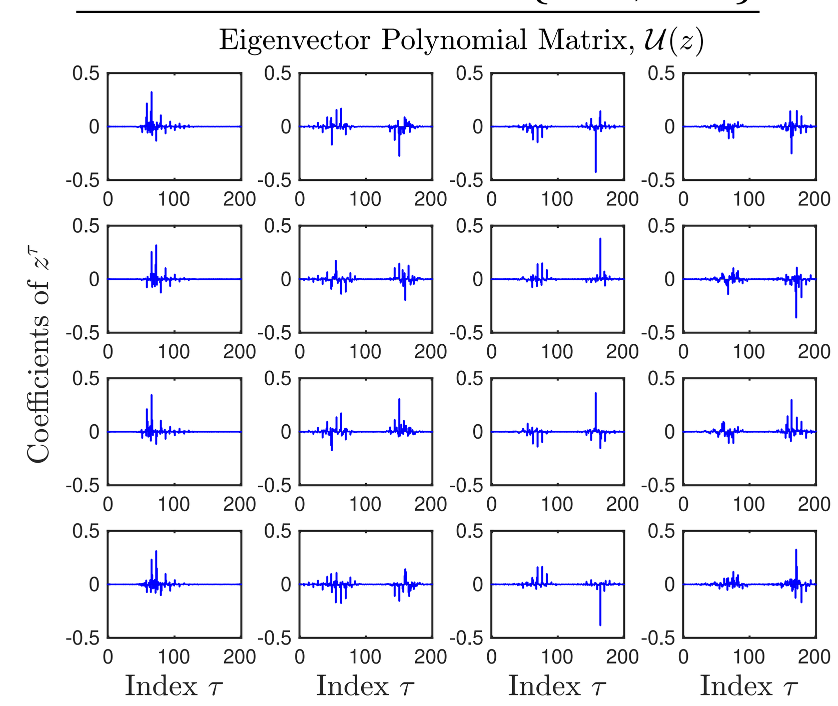
Filter length statistics
- Min: 17, Max: 372, Mean: 114

Listening examples are available [4].

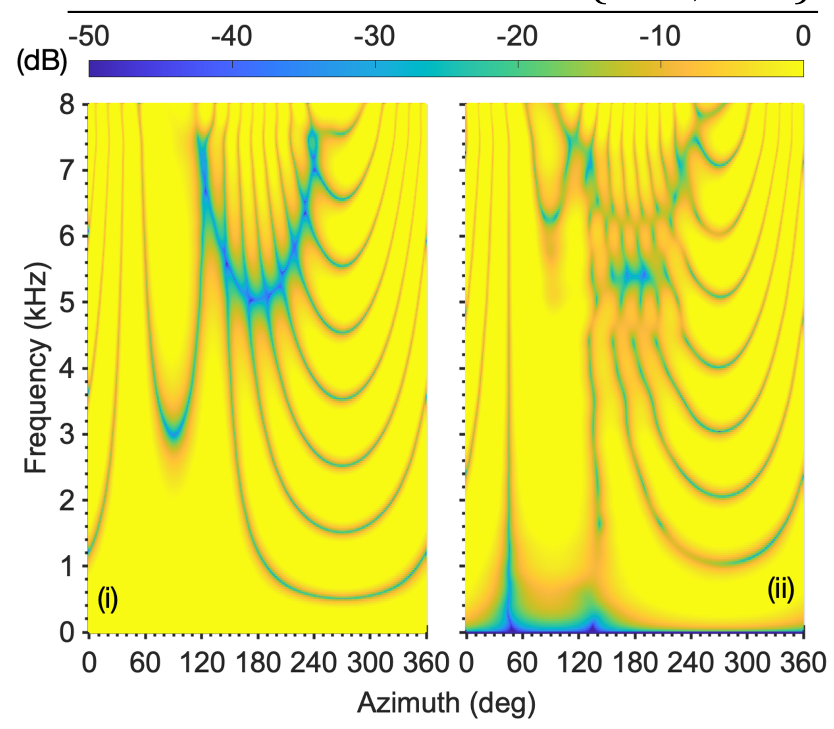
Simulation Results for Separation of Two Speakers

	S1 ($\phi_1 = 45°$)		S2 ($\phi_2 = 50°$)	
Algorithm	ΔSTOI	ΔSIR (dB)	ΔSTOI	SIR (dB)
PEVD $\{45°, 50°\}$	0.002	- 0.034	**0.204**	15.752
PEVD $\{50°, 45°\}$	**0.123**	16.703	0.004	0.247
MVDR	0.113	13.487	0.186	12.435
LCMV	0.047	**19.986**	0.156	**23.522**

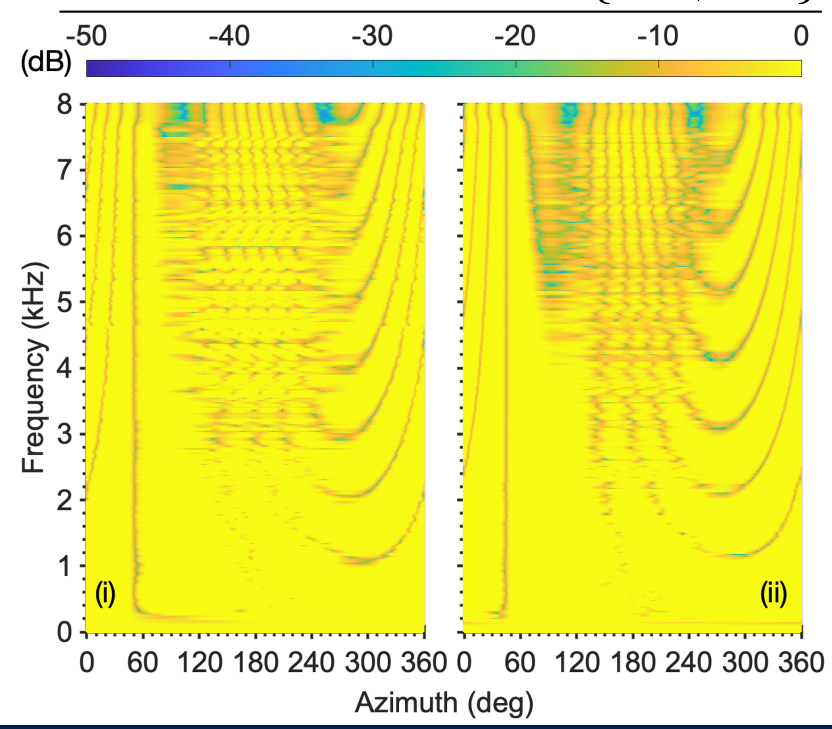
PEVD filterbank for $\{45°, 50°\}$



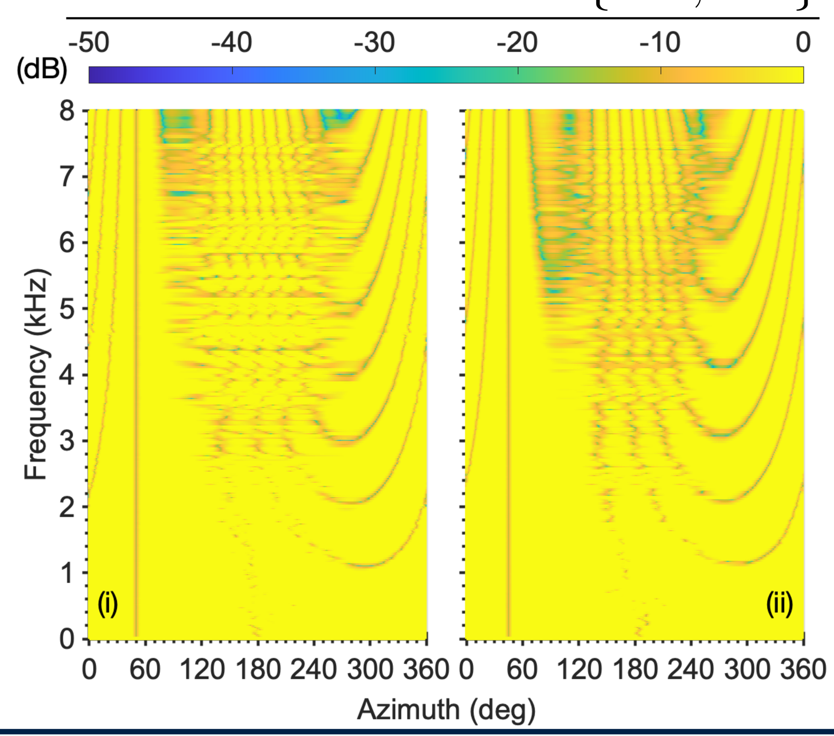
PEVD beamformer for $\{45°, 50°\}$



MVDR beamformer for $\{45°, 50°\}$



LCMV beamformer for $\{45°, 50°\}$



References

[1] S. Weiss, J. Pestana, and I. K. Proudler, "On the existence and uniqueness of the eigenvalue decomposition of a para-Hermitian matrix," *IEEE Transactions Signal Processing*, vol. 66, no. 10, pp. 2659–2672, May 2018.

[2] A. H. Moore, *Free field hearing aid array*, 2020. [Online]. Available: `https://github.com/ImperialCollegeLondon/sap-elobes-microphone-arrays`.

[3] E. A. P. Habets, "Room impulse response generator," Technische Universiteit Eindhoven (TU/e), Tech. Rep. 2006.

[4] V. W. Neo, *PEVD fixed beamformer demo*, Apr. 2022. [Online]. Available: `https://vwn09.github.io/research/pevd-beamformer-iwaenc`.